

PRACTICE Quantum Detanglers

Division C

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(Answer Key at END)

Constants

Speed of Light	c	3e8 m/s
Mass of Electron	m_e	9.11e-31 kg 0.511 MeV/ c^2
Mass of Proton	m_p	1.673e-27 kg 938.3 MeV/ c^2
Mass of Neutron	m_n	1.675e-27 kg 939.6 MeV/ c^2
Electron charge	e	1.6e-19 C
Electron Volt	eV	1.6e-19 J
Planck's Constant	h	6.626e-34 J s 4.135e-15 eV s
Reduced Planck's Constant	\hbar	1.054e-34 J s 0.658e-15 eV s

Quantum Mechanics

Note: Wave mechanics problems are classically done with integrals, but for the scope of this exam, a simple summation over a discrete interval will suffice.

For example, if one is using Born's rule to calculate the probability that a particle with the wavefunction is within $2 \leq x \leq 5$, the probability is , using discrete, integer values of x .

Henrietta knows some of the differences between classical and quantum mechanics, she wonders why classical mechanics works in macroscopic systems. What principle states that quantum systems reproduce classical systems when dealing with large quantum numbers, such as energies and number of particles?

- a) Reduction principle
- b) Correspondence principle
- c) Law of large numbers
- d) Simplification principle

Henrietta knows that she can't determine the momentum and position with certainty of a quantum particle. She thinks that this is due to experimental uncertainty and that an unobservable, definite momentum and position of the particle exists. What theorem refers to this idea?

- a) Dynamic variable theorem
- b) Einstein-Podolsky-Rosen causality theorem
- c) Uncertainty theorem
- d) De Broglie-Bohm theorem

According to the Copenhagen interpretation, can particle and wave nature be observed simultaneously in a system?

- a) Yes
- b) No

Isaac is heating up a perfect black body. He discovers that the intensity of the emitted black-body radiation at lower frequencies is proportional to the temperature. He wants to use an equation to predict the intensity of his black-body radiation at all wavelengths based on temperature.

Which of the below equations does this? (Circle all that apply)

- a) Rayleigh-Jeans Law
- b) Wien's Displacement Law
- c) Einstein-Planck Law
- d) Bohr's Radiation Law
- e) Stefan-Boltzmann Law

Isaac chooses the Rayleigh-Jeans Law to model his black-body data. He notices that the equation poorly models the intensity at high frequencies. What is the error between his experimental and theoretical data?

- a) The experimental data is much higher than the theoretical model.
- b) The experimental data is both higher and lower than the theoretical model.
- c) The experimental data is much lower than the theoretical model.

What is the name of this historical physics problem between experiment and theory?

- a) Infinite Intensity Problem
- b) Light Conservation Postulate
- c) Ultraviolet Catastrophe
- d) Ehrenfest Problem

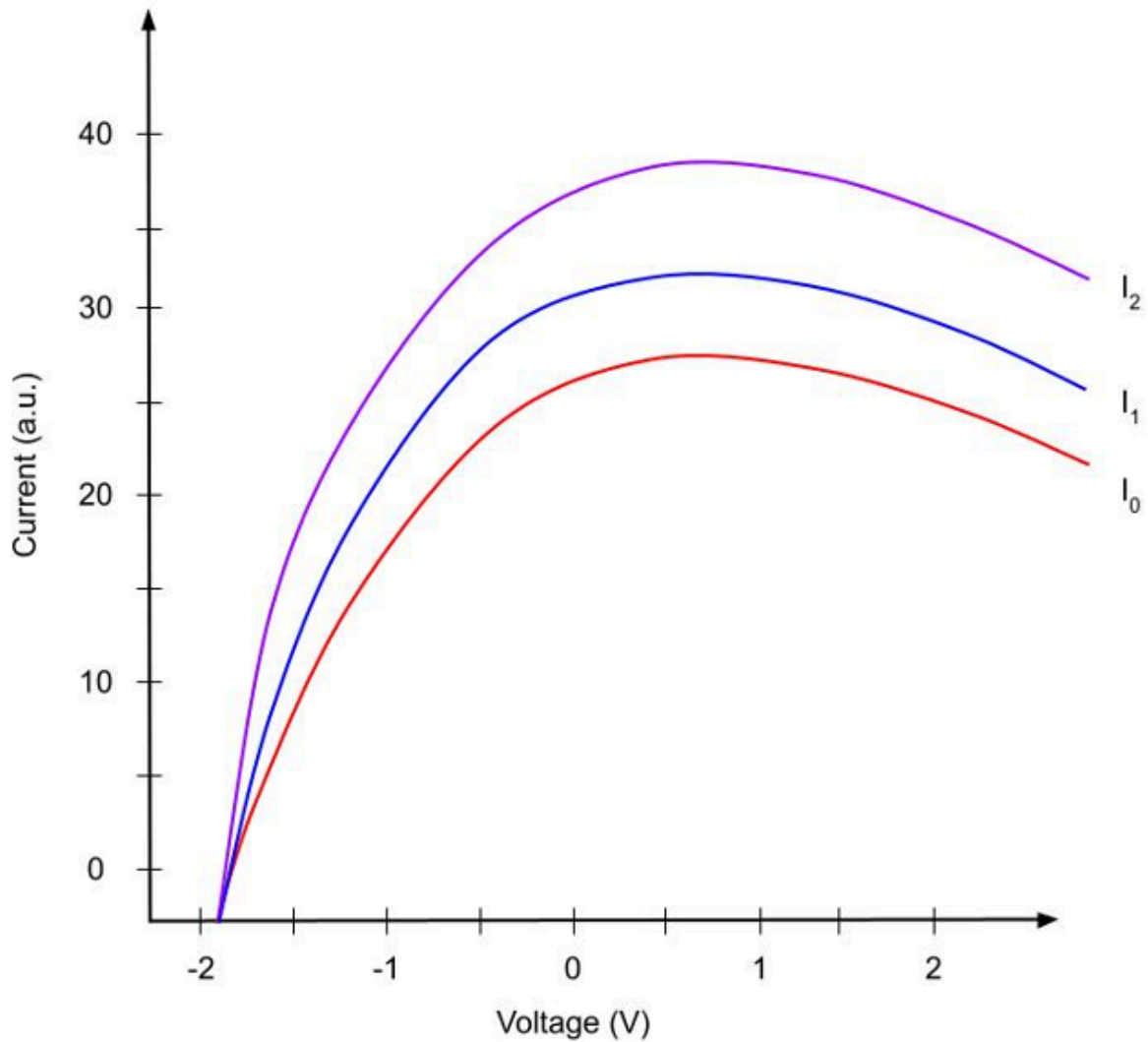
Who solved this problem?

- a) Niels Bohr
- b) Max Planck
- c) Max Born
- d) Albert Einstein

Isaac measures the intensity at a frequency of 10^{10} Hz. What is the difference in energy levels at this frequency?

- a) 6.626×10^{-24} J
- b) 3.923 eV
- c) 6.032×10^{-20} J
- d) 0.345 eV

Irene learned about the photoelectric effect in class and wanted to prove it for herself. She decides to recreate the Lenard experiment. Shining a monochromatic light of wavelength 325nm on the cathode, she gets the following photoelectric current vs. voltage graph.



What is the stopping potential of this experimental setup?

- a) 0.5V
- b) -2V
- c) 1.9V
- d) 1V

List the light intensity of the three curves from greatest to least.

- a) I_0, I_1, I_2
- b) I_1, I_0, I_2
- c) I_2, I_0, I_1
- d) I_2, I_1, I_0

What is the work function of the unknown cathode metal?

- a) 1.900 eV
- b) 1.915 eV
- c) 3.815 eV
- d) 0.190 eV

Carl wants to prove that angular momentum (spin) is quantized. What property arises from spin in subatomic particles?

- a) Electric dipole
- b) Magnetic monopole
- c) Electric moment
- d) Magnetic moment

Which of the particles below in the Standard model has half integer spin (circle all that apply)?

- a) Higgs-boson
- b) Tau Neutrino
- c) Gluon
- d) Down Quark
- e) Electron

Micheal wants to determine the value for the quantization of charge (elementary charge). What historical experiment should he replicate?

- a) Faraday's Experiment
- b) Thomson's Experiment
- c) Michelson-Morley Experiment
- d) Millikin's Experiment

Suppose that the energy of an electron in an infinite square well at $n = 3$ is $E_3 = 0.0418$ eV and the energy at $n = 4$ is $E_4 = 0.0743$ eV. What is the length of the infinite square well?

- a) 10 nm
- b) 90 nm
- c) 100 nm
- d) 9 nm

Suppose that a proton is trapped in an infinite square well with unknown length at an unknown energy level, n . When the proton falls from energy level n to $n-1$, a photon of frequency $8.670e8$ Hz is emitted. When the proton falls from $n-1$ to $n-2$, a photon of frequency $6.193e8$ Hz is emitted. What is n ?

- a) 3
- b) 4
- c) 5

- d) 6

What is the length of the infinite square well?

- a) 10 nm
b) 20 nm
c) 30 nm
d) 40 nm

Consider an unknown molecule that acts as a quantum harmonic oscillator with a vibrational angular frequency of $\omega = 1.8 \times 10^{17}$ rad/s. What is the shortest wavelength of light that can be emitted from the $n = 5$ vibrational modes?

- a) 2.095 nm
b) 10.476 nm
c) 1.905 nm
d) 6.985 nm

What is the normalization factor for the wavefunction $\Psi(x) = e^{-2x}$, $0 \leq x \leq 5$?

- a) 1
b) 0.864
c) 6.391
d) 0.982

Consider a particle with the wavefunction $\Psi(x) = 10e^{-7x} + 2e^{-8x} + 3e^{-2x}$. Does this particle's position have more or less spread than that of the particle mentioned previously?

- a) More
b) Less

Jocelyn is observing a pion in a lab, which is a spin zero particle. Due to the unstable nature of pions, it soon decays into a muon and muon neutrino, which have half integer spins. Assuming that neither muon nor muon neutrino are initially observed, are they entangled?

- a) Yes
b) No

Jocelyn measures the spin of the muon and observes that it is spin down. What can Jocelyn say about the muon neutrino?

- a) It is spin down.
b) It is spin up.
c) Jocelyn cannot infer anything about the spin of the muon neutrino.

Which of the following historical physics laws does not obey locality?

- a) Special Relativity
- b) Coulomb's Law
- c) General Relativity
- d) Maxwell's Equations

Which of the following are critical assumptions made when deriving Bell's Inequality? (Circle all that apply)

- a) Realism
- b) Locality
- c) Causality
- d) Complementarianism

Quantum Biology:

1. Briefly describe quantum tunneling and provide an example of how it could influence a biological reaction.
 2. Explain some challenges in harnessing quantum effects for targeted drug delivery systems.
 3. How are quantum dots used in biological imaging and diagnostics? List a type of quantum dot and its possible application
 4. Explain the concept of relaxation times in the context of MRI.
1. Photosynthetic organisms may utilize quantum effects to optimize which biological process?
 - a) DNA replication
 - b) Protein synthesis
 - c) Energy transfer
 - d) Cellular respiration
 2. Which of the following statements about quantum tunneling is correct?
 - a) It requires a large input of energy
 - b) It involves particles colliding and exchanging energy
 - c) It allows particles to pass through energy barriers without meeting classical energy requirements
 - d) It only occurs in high-energy environments
 3. Which of the following biological processes is thought to be influenced by quantum effects in the context of magnetoreception?

- a) Muscle contraction
- b) Cellular respiration
- c) Vision
- d) Olfaction

Quantum Computing:

Which of the following describes the effective function of a CNOT gate?

- a. If the first qubit is in state $|0\rangle$, perform a Pauli-X gate on the second qubit. If the first qubit is in state $|1\rangle$, leave the second qubit as is.
- b. If the first qubit is in state $|0\rangle$, leave the second qubit as is. If the first qubit is in state $|1\rangle$, perform a Pauli-X gate on the second qubit.
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Which of the following gates, when applied twice, brings the system back to its original state?

- a. Hadamard Gate
- b. Pauli-Z Gate
- c. Pauli-Y Gate
- d. Phase Gate

The following circuit is equivalent to what quantum gate?

- a. Phase Gate
- b. Controlled Z Gate
- c. Pauli X Gate
- d. Hadamard Gate

What is the rank of a CCNOT gate?

- a. 2
- b. 4
- c. 8
- d. 12

What is the primary application of the Deutsch-Jozsa Algorithm?

- a. Quantum Teleportation
- b. Solving systems of linear equations in constant time
- c. Identifying whether a function is constant or balanced
- d. Identifying the prime factors of large positive integers

What is the role of the controlled-NOT (CNOT) gate in quantum computing?

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- b. Performs a Hadamard transformation on the target qubit.
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What is the probability of a state in the following circuit?

What problem does quantum counting algorithm address?

- a. Determines the prime factors of large integers
- b. Counts the number of solutions to a specified problem
- c. Efficiently searches a sorted database
- d. Simulates quantum circuits

Which of the following quantum gates is not its own inverse?

- a. Hadamard Gate
- b. Pauli Z Gate
- c. Phase Gate
- d. CNOT Gate

Answer Key BELOW

ANSWER KEY

Quantum Mechanics

Note: Wave mechanics problems are classically done with integrals, but for the scope of this exam, a simple summation over a discrete interval will suffice.

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Quantum Biology:

Briefly describe quantum tunneling and provide an example of how it could influence a biological reaction.

Quantum tunneling is a phenomenon in quantum mechanics where particles can pass through energy barriers that would be classically impassable. It could lower enzyme catalysis activation energies required for the reaction to occur. Make sure to earn points to elaborate on the quantum phenomena and how tunneling actually reduces activation energy.

Explain some challenges in harnessing quantum effects for targeted drug delivery systems.

Biocompatibility, stability, specific site targeting, scalability and manufacturing. These are all good ideas and terms that can be used to describe this. Points will be rewarded on how well you connect these ideas with quantum effects such as superposition, or entanglement.

How are quantum dots used in biological imaging and diagnostics? List a type of quantum dot and its possible application

Explain the quantum effects surrounding quantum dots -- basically how synthetic semiconductor nanoparticles can be used in imaging due to quantum confinement effects -- explain how this allows for a wide wavelength of colors to be produced. Carbon QDs are a type of qd used in cancer treatment by finding and then destroying tumor cells.

Explain the concept of relaxation times in the context of MRI.

In the context of MRI (Magnetic Resonance Imaging), relaxation times refer to the timescales associated with the return of excited nuclear spins to their equilibrium states. MRI relies on the behavior of atomic nuclei, specifically hydrogen nuclei (protons), in a strong magnetic field.

There are two relaxation times:

Longitudinal Relaxation Time: T1 is the time it takes for the nuclear spins to recover to their equilibrium state parallel to the external magnetic field after being perturbed by radiofrequency pulses.

Transverse Relaxation Time: T2 is the time it takes for the transverse magnetization to decay after the radiofrequency pulse is turned off. During excitation, the protons rotate around the axis of the magnetic field and produce a transverse magnetization.

Photosynthetic organisms may utilize quantum effects to optimize which biological process?

- a) DNA replication
- b) Protein synthesis
- c) Energy transfer**
- d) Cellular respiration

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3) Which of the following gates, when applied twice, brings the system back to its original state?

- a. **Hadamard Gate**
- b. Pauli-Z Gate
- c. Pauli-Y Gate
- d. Phase Gate

5) The following circuit is equivalent to what quantum gate?

- a. Phase Gate
- b. Controlled Z Gate
- c. **Pauli X Gate**
- d. Hadamard Gate

6) What is the rank of a CCNOT gate?

- a. 2
- b. 4
- c. **8**
- d. 12

7) What is the primary application of the Deutsch-Jozsa Algorithm?

- a. Quantum Teleportation
- b. Solving systems of linear equations in constant time
- c. **Identifying whether a function is constant or balanced**
- d. Identifying the prime factors of large positive integers

8) What is the role of the controlled-NOT (CNOT) gate in quantum computing?

- a. **Flips the state of the target qubit if the control qubit is in the $|1\rangle$ state.**
- b. Performs a Hadamard transformation on the target qubit.
- c. Measures the state of the control qubit and applies the result to the target qubit.
- d. Swaps the states of the control and target qubits.

9) What is the probability of a state $|00\rangle$ in the following circuit?

Answer. 0

10) What problem does quantum counting algorithm address?

- a. Determines the prime factors of large integers
- b. **Counts the number of solutions to a specified problem**

- c. Efficiently searches a sorted database
- d. Simulates quantum circuits

11) Which of the following quantum gates is not its own inverse?

- a. Hadamard Gate
- b. Pauli Z Gate
- c. **Phase Gate**
- d. CNOT Gate